On Equation of Motion for Arbitrarily Shaped Particle under Action of Electromagnetic Radiation

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Abstract. Arguments of astronomers against equation of motion for arbitrarily shaped particle under action of electromagnetic radiation are discussed. Each of the arguments is commented in detail from the point of view of the required physics. It is shown that the arguments of astronomers, including referees in several astronomical and astrophysical journals, are unacceptable from the physical point of view. Detail explanations should help astronomers in better physical understanding of the equation of motion. Relativistically covariant equation of motion for real dust particle under the action of electromagnetic radiation is derived. The particle is neutral in charge. Equation of motion is expressed in terms of particle's optical properties, standardly used in optics for stationary particles.

Key words: relativity theory, electromagnetic radiation, cosmic dust

1. Introduction

Astronomers each year publish several papers on orbital evolution of cosmic (dust) particles which consider interaction of the particles with electromagnetic radiation. This is taken in the form of the Poynting-Robertson effect (P-R effect; Robertson 1937). However, P-R effect corresponds to a very special form of interaction between the particle electromagnetic radiation (see Eqs. (120), (122) in Klačka 1992a). Since real particles do not fulfil this special form of interaction, one needs to have more general equation of motion in disposal. This is presented in Klačka (2000a). Later on, more simple derivations were presented: Klačka (2000b), Klačka and Kocifaj (2001a), see also review paper Klačka (2001b). Applications of the equation of motion can be found in Klačka (2000c), Klačka and Kocifaj (2001a), Klačka and Kocifaj (2001b), Klačka and Kocifaj (2002), Kocifaj and Klačka (2002a, 2002b).

Our experience shows that astronomers do not understand the physics of the equation of motion. This is the reason why we have decided to present arguments against the equation of motion and, of course, to give detail explanations why the arguments are physically incorrect. We can say that this paper is a continuation of the paper Klačka (1993).

2. Arguments and answers

This section presents arguments/statements of astronomers against the equation of motion (since the year 1998) and our point of view.

2.1. Argument/statement 1

Astronomers do not need any more general equation of motion – they are satisfied with the Poynting-Robertson effect.

Answer:

Scattering of light, electromagnetic radiation, on arbitrarily shaped (dust) particles with various optical properties may significantly differ from that required by the P-R effect: compare Klačka (1992a) and Kocifaj and Klačka (1999). As a consequence, orbital evolution of real particle practically always differs from that corresponding to the P-R effect: Kocifaj et al. (2000), Klačka and Kocifaj (2001a), Klačka and Kocifaj (2001b), Klačka and Kocifaj (2002), Kocifaj and Klačka (2002a, 2002b).

2.2. Argument/statement 2

Orthonormality of unit vectors used in the scattering theory holds also in the frame of reference in which particle moves with velocity $\boldsymbol{v}$ *(also for* $v \neq 0$*).*

Answer:

Although this access was used in Klačka (1994), Klačka and Kocifaj (1994) and Kocifaj et al. (2000), it is not physically correct. Unit vectors must be orthonormal in the proper frame of reference (the rest frame of the particle, i. e. a moving frame relative to the source of electromagnetic radiation). The requirement that "final equation of motion has to be written in a covariant form" offers only one possibility: vectors are not orthonormal in any other frame of reference. And physical interpretation is also evident – aberration of light (Klačka 2000a, 2000b). Thus, the statement about the conservation of normality of vectors under Lorentz transformation is nonphysical.

However, experience shows that astronomers do not accept this general argument. Thus, we will present a simple example which shows that access of Klačka (2000a) is correct (at least for the simple example).

EXAMPLE:

Let us consider a plane mirror moving (at a given moment) along x-axis (system S) with velocity $\boldsymbol{v} = (v, 0, 0)$, $v > 0$; the mirror is perpendicular to the x-axis (the plane of the mirror is parallel to the yz-plane). A beam of incident (hitting) photons is characterized by unit vector $\boldsymbol{S}' = (\cos\theta', \sin\theta', 0)$ in the proper frame (primed quantities) of the mirror. Reflected beam is described by the unit vector $\boldsymbol{e}' = (- \cos\theta', \sin\theta', 0)$ (in the proper frame S').

The problem is: Find equation of motion of the mirror in the frame of reference S.

SOLUTION 1: trivial manner

Consider one photon (frequency f') in the proper frame of the mirror. Since the directions (and orientations) of the incident and outgoing photons are characterized by

$$\begin{aligned} \boldsymbol{S}' &= (+\cos\theta', \sin\theta', 0) \ , \\ \boldsymbol{e}' &= (-\cos\theta', \sin\theta', 0) \ , \end{aligned} \tag{1}$$

we can immediately write

$$\begin{aligned} p_i^{'\mu} &= \frac{h\,f'}{c}\ (1, +\cos\theta', \sin\theta', 0) \ , \\ p_o^{'\mu} &= \frac{h\,f'}{c}\ (1, -\cos\theta', \sin\theta', 0) \ , \end{aligned} \tag{2}$$

for the four-momentum of the photon before interaction with the mirror and after the interaction.

As a consequence, the mirror obtains four-momentum

$$p^{'\mu} = p_i^{'\mu} - p_o^{'\mu} = \frac{h\,f'}{c}\ (0, 2\cos\theta', 0, 0) \ . \tag{3}$$

Application of the special Lorentz transformation to Eq. (3) yields

$$p^{\mu} = \frac{h\ f'}{c}\ 2\ \gamma\ \left(\cos\theta'\right)\ \left(\beta, 1, 0, 0\right)\ , \tag{4}$$

where, as standardly abbreviated,

$$\gamma = 1\ /\ \sqrt{1\ -\ \beta^2}\ \ ;\quad \beta = v\ /\ c\ . \tag{5}$$

On the basis of Eq. (4), we can immediately write equation of motion of the mirror

$$\frac{dp^{\mu}}{d\tau} = \frac{E'_i}{c}\ 2\ \gamma\ \left(\cos\theta'\right)\ \left(\beta, 1, 0, 0\right)\ , \tag{6}$$

where E'_i is the total energy (per unit time) of the incident radiation measured in the proper frame of reference.

SOLUTION 2: application of general theory of Klačka (2000a)

We have to choose orthonormal vectors in the systems S': we will use $\boldsymbol{S}'$ and $\boldsymbol{e}'_1$ and one can easily find

$$\begin{aligned} \boldsymbol{S}'\ &=\ (+\ \cos\theta', \sin\theta', 0)\ , \\ \boldsymbol{e}'_1\ &=\ (-\ \sin\theta', \cos\theta', 0)\ . \end{aligned} \tag{7}$$

We have to write ($Q'_2 = 0$)

$$\boldsymbol{p}' = \frac{h\ f'}{c}\ \left(Q'_R\ \boldsymbol{S}'\ +\ Q'_1\ \boldsymbol{e}'_1\right)\ . \tag{8}$$

On the basis of Eqs. (3), (7) and (8) we have

$$Q'_R = 2\ \left(\cos\theta'\right)^2\ \ ,\quad Q'_1 = -\ 2\ \left(\sin\theta'\right)\ \left(\cos\theta'\right)\ ;\quad \left(Q'_2 = 0\right)\ . \tag{9}$$

Other prescription yields

$$\begin{aligned} b_i^0\ &=\ \gamma\ \left(1\ +\ \boldsymbol{v}\cdot\boldsymbol{S}'/c\right) = \gamma\left(1\ +\ \beta\ \cos\theta'\right)\ , \\ \boldsymbol{b}_i\ &=\ \boldsymbol{S}'\ +\ \left[\left(\gamma\ -\ 1\right)\ \boldsymbol{v}\cdot\boldsymbol{S}'/\boldsymbol{v}^2\ +\ \gamma/c\right]\ \boldsymbol{v} = \left(\gamma\ \cos\theta'\ +\ \gamma\ \beta, \sin\theta', 0\right)\ , \end{aligned} \tag{10}$$

$$\begin{aligned} b_1^0\ &=\ \gamma\ \left(1\ +\ \boldsymbol{v}\cdot\boldsymbol{e}'_1/c\right) = \gamma\left(1\ -\ \beta\ \sin\theta'\right)\ , \\ \boldsymbol{b}_1\ &=\ \boldsymbol{e}'_1\ +\ \left[\left(\gamma\ -\ 1\right)\ \boldsymbol{v}\cdot\boldsymbol{e}'_1/\boldsymbol{v}^2\ +\ \gamma/c\right]\ \boldsymbol{v} = \left(-\ \gamma\ \sin\theta'\ +\ \gamma\ \beta, \cos\theta', 0\right)\ . \end{aligned} \tag{11}$$

Inserting Eqs. (9) – (11) into Eq. (28) of Klačka (2000a), one obtains

$$\begin{aligned} \frac{dp^{\mu}}{d\tau}\ &=\ \frac{E'_i}{c}\ \left\{\left[2\ \left(\cos\theta'\right)^2\right]\ \left(b_i^{\mu}\ -\ \beta^{\mu}\right)\ +\ \left[-\ 2\ \left(\sin\theta'\right)\ \left(\cos\theta'\right)\right]\ \left(b_1^{\mu}\ -\ \beta^{\mu}\right)\right\} \\ &=\ \frac{E'_i}{c}\ 2\ \gamma\ \left(\cos\theta'\right)\ \left(\beta, 1, 0, 0\right)\ . \end{aligned} \tag{12}$$

COMPARISON:

Unit vectors $\boldsymbol{S}'$ and $\boldsymbol{e}_1'$ are used according to Klačka (2000a). They lead to correct results. The unit vectors are orthonormal in the system S'. How does the situation look in the system S?

$$\boldsymbol{S} = \frac{1}{w'} \left\{ \boldsymbol{S}' + \left[(\gamma - 1) \, \boldsymbol{v} \cdot \boldsymbol{S}' / \boldsymbol{v}^2 + \gamma / c \right] \, \boldsymbol{v} \right\} ,$$
$$w' = \gamma \, (1 + \boldsymbol{v} \cdot \boldsymbol{S}' \, / \, c) \, , \tag{13}$$

and analogous equation holds for vector $\boldsymbol{e}_1$. Inserting Eqs. (7), one obtains

$$\boldsymbol{S} = \left\{ \frac{\cos\theta' + \beta}{1 + \beta \, \cos\theta'}, \frac{\sin\theta'}{\gamma \, (1 + \beta \, \cos\theta')}, 0 \right\} ,$$
$$\boldsymbol{e}_1 = \left\{ \frac{- \, \sin\theta' + \beta}{1 - \beta \, \sin\theta'}, \frac{\cos\theta'}{\gamma \, (1 - \beta \, \sin\theta')}, 0 \right\} . \tag{14}$$

It can be easily verified that scalar product of these two vectors is nonzero, in general, even to the first order in β:

$$\boldsymbol{S} \cdot \boldsymbol{e}_1 \approx \beta \, (\cos\theta' - \sin\theta') \, . \tag{15}$$

SOLUTION 3: application of general theory of $G^{\mu \ \nu} \ b_{i \ \nu}$ (Klačka 2001a).

We will use equation of motion in the form:

$$\frac{d\boldsymbol{p}'}{d\tau} = \frac{E_i'}{c} \, Q' \, \boldsymbol{S}' \, . \tag{16}$$

Calculations yield

$$Q' = Q_i' - Q_r' = diag(1, 1, 1) - diag(- 1, 1, 1) = diag(2, 0, 0) \, . \tag{17}$$

Moreover, expression for $\boldsymbol{S}$ presented in Eq. (14) yields

$$(Q' \boldsymbol{S})^T = 2 \, \left(\frac{\cos\theta' + \beta}{1 + \beta \, \cos\theta'}, 0, 0 \right)$$
$$(Q' \boldsymbol{\beta})^T = 2 \, (\beta, 0, 0) \, , \tag{18}$$

$$\beta^T (Q' \boldsymbol{S}) = 2 \, \beta \, (\cos\theta' + \beta) \, / \, (1 + \beta \, \cos\theta')$$
$$\beta^T (Q' \boldsymbol{\beta}) = 2 \, \beta^2 \, . \tag{19}$$

Inserting into equation

$$\frac{dp^\mu}{d\tau} = \frac{E'_i}{c} G^{\mu\ \nu} b_{i\ \nu} \ , \tag{20}$$

one obtains

$$\frac{dp^\mu}{d\tau} = \frac{E'_i}{c} 2 \ \gamma \ (\cos \theta') \ (\beta, 1, 0, 0) \ . \tag{21}$$

REMARK: Inserting $E'_i = w^2 \ S \ A' \ \cos \theta'$ into Eq. (6) (or Eqs. (12), (21)) and using Eq. (14) for the purpose of obtaining $\cos \theta' = (\cos \theta - \beta)/(1 - \beta \ \cos \theta)$, one easily obtains:

i) $dE/d\tau = 2 \ \gamma^3 \ S \ A' \ (\cos \theta - \beta)^2 \ \beta$; using definition of radiation pressure $dE/dt \equiv P \ v \ A'$, we have $P = 2 \ (S \ / \ c) \ (\cos \theta - \beta)^2 \ / \ (1 - \beta^2)$, or,

ii) $dp/d\tau = 2 \ \gamma^3 \ (S \ A' \ / \ c) \ (\cos \theta - \beta)^2$; using definition of radiation pressure $P \equiv (dp/dt) \ / \ A'$, we have $P = 2 \ (S \ / \ c) \ (\cos \theta - \beta)^2 \ / \ (1 - \beta^2)$.

Result for P is consistent with result presented in Einstein (1905).

2.3. Argument/statement 3

The assumption is that energy E' of the particle is unchanged: the energy of the incoming radiation equals the energy of the outgoing radiation, per unit time. What this means is that the particle does not heat up. Fine, as such, but wait. A physical explanation here involving the Poynting-Robertson effect has the particle losing orbital energy due to re-radiation effects, and so the size of its orbit reduces. So how does it work, physically?

Answer:

Yes, $dE' \ / \ d\tau = 0$ corresponds to conservation of mass of the particle. However, the relation for energy holds in the proper reference frame of the particle (rest frame of the particle), only. Lorentz transformation yields that energy changes in other reference frames, e. g. in the rest frame of the source of radiation. The special case $Q'_1 = Q'_2 = 0$ in Klačka (2000a – e. g., Eqs. (7), (23)) – corresponding to the P-R effect – yields that $dE/dt \neq 0$ and the corresponding change of semi-major axis is given by Eq. (14) in Klačka (1992b).

2.4. Argument/statement 4

Where does the "really [realistically?] shaped particle come into it?"

Answer:

Effective factors Q_R, Q_1, Q_2 (see Eq. (7) in Klačka 2000a) take into account optical characteristics of the particle. They can be calculated using, e. g., discrete dipole approximation – see, e. g., Draine and Weingartner (1996).

2.5. Argument/statement 5

$dE' \ / \ d\tau = 0$: If a momentum is transferred to the particle there should be a corresponding gain in kinetic energy. In reality this term will depend upon the particle's albedo etc.

Answer:

The statement $dE' \ / \ d\tau = 0$ was discussed in section 2.3 – conservation of particle's mass. Even within a Newtonian physics we have $T = m \ v^2/2$, $dT/dt = m \ \boldsymbol{v} \cdot \dot{\boldsymbol{v}}$ and this yields $dT'/dt = 0$ in the proper frame of the particle, although $d\boldsymbol{p'}/dt \neq 0$. In reality $dE/dt \neq 0$ in the rest frame of the source and dE/dt depends on optical properties of the particle (see sections 2.3 and 2.4). (It has no sense to use the term "particle's albedo" if the size of the particle is comparable to the wavelength of the incident electromagnetic radiation, when measured in the proper frame of the particle.)

2.6. Argument/statement 6

The problem is of great general interest and (at least at the Newtonian level) has been treated intensively in the literature.

Answer:

Since the time of Maxwell and Einstein (1905) we know that the discussed problem cannot be treated at the Newtonian level. In reality, motion of cosmic dust grains is standardly described by the P-R effect (of course, it cannot be understood at the Newtonian level). P-R effect is a very special case of the general equation of motion (Eq. (24) in Klačka 2000a).

2.7. Argument/statement 7

A real astronomical problem is oversimplified and the resulting equations are not really useful.

Answer: Papers Klačka and Kocifaj (2001a, 2001b) show important difference between orbital motion obtained on the basis of Eq. (24) in Klačka (2000a) and the P-R effect.

P-R effect was used for six decades (up to now) and there was no astronomical protest that "it is not really useful".

3. Conclusion

We have discussed some statements of astronomers as for more general equation of motion than that given by the P-R effect. We have tried to "stand up for" the general equation of motion, as we have presented it for the first time in Eq. (24) in Klačka (2000a) and in a more simple form in Klačka (2000a), in Klačka and Kocifaj (2001a). We have shown that our more general equation of motion reduces to the two cases discussed earlier in literature: Einstein (1905), Robertson (1937). Any correct equation of motion has to be consistent with the results of Einstein (1905) and Robertson (1937).

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